

ATTAINMENT'S  
**EXPLORE**  
**Geometry**

**INSTRUCTOR'S  
GUIDE**

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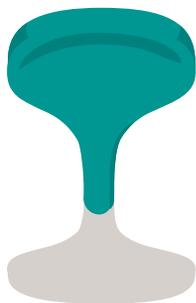
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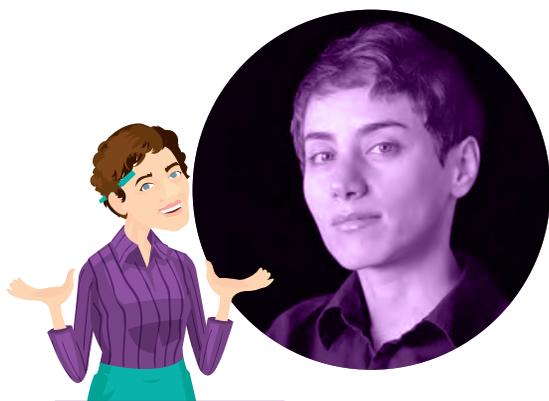
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# A Little about Professor Maryam and Professor Euclid

**Fun Fact:** The two professors exploring geometry with us are based on real-life mathematicians!



**Maryam Mirzakhani (May 11, 1977—July 14, 2017)** was an Iranian mathematician and professor at Stanford University. She was born in Tehran, Iran. As a schoolgirl, Maryam competed in many math competitions and won gold medals for first place. She continued her math studies in college, earning her a Bachelor of Science degree from Sharif University of Technology and then a PhD from Harvard University. Maryam's research and achievements as a professor won her many prestigious awards in the field of mathematics with a focus in geometry. In 2014, while teaching at Stanford University, she became the first woman and first Iranian to be awarded a Fields Medal, a special award given to mathematicians every four years for outstanding work in math.

Sadly, Maryam died from cancer in 2017. She is remembered as an exceptional geometer, among many other things.



**Euclid (325 BCE—270 BCE)** was an ancient Greek mathematician from Alexandria, Greece. He is known as the Father of Geometry! There aren't many specifics known about his life; however, his writing about geometry, titled *The Elements*, has been commonly used as a geometry textbook for the last 2,000 years! In addition to *The Elements*, he wrote several other books that are still used by geometers to this day.

# FOREWARD

English learners (ELs), students with disabilities, and ELs with disabilities or extensive support needs face many challenges in American educational systems. Not only are they navigating these systems with unique learning needs; but some are doing so while acquiring a second language, others are adjusting to new cultural norms, and some experience challenges with communication including the aforementioned. To assist in the increased academic and functional performance of these groups, researchers have illustrated and advocated for the continued use of instructional approaches that combine a wide-range research-based strategies and frameworks as a way to further individualize instruction depending on student needs (e.g., Cook & Rao, 2018; Lopes-Murphy, 2012; Rivera et al., 2019; Roa et al., 2017). One of these approaches is the Universal Design for Learning (UDL; CAST, 2018). UDL places emphasis on three prominent guidelines for increasing academic performance. First, the framework suggests that students should be provided with multiple means of Engagement. This can be achieved by developing lessons that provide varied options that will captivate student interest, increase collaboration amongst students, and promote self-reflection. Secondly, UDL seeks to ensure that lessons are taught in a way that promote multiple means of Representation. In other words, students are given options in how they view information that is presented and are supported in ways to help promote understanding across languages. Finally, the UDL framework seeks to give students multiple means of Action and Expression. This final component encourages educators to seek different ways in which students can demonstrate their knowledge through, for instance, varied expressive and communicative intents.

For ELs, UDL is an additional layered framework that can be beneficial in the acquisition of academic language. Lopes-Murphy (2012) argued that ELs need scaffolded language supports that help connect prior knowledge with new information. In addition, students need opportunities to engage in academic language with other learners across multiple settings. Students with the most diverse learning needs need diverse solutions. This is where the addition of UDL is beneficial. Not only does it provide instructional flexibility, but it challenges educators to think carefully about how to increase accessibility to content while also focusing on language development. While UDL is important, Rivera et al. (2016) adds that ELs with extensive support needs should be given multiple opportunities to actively participate in lessons taught and should be granted the same access to the general curriculum as their peers, despite complex language or communication needs. The same can be said of monolingual students with extensive support needs.

While research in using UDL for students with disabilities has been established (e.g., Cook & Rao, 2018; Rao et al., 2017), its application is still emerging regarding ELs with extensive support needs (Rivera et al., 2019), and examples of applying existing strategies and instructional frameworks are still somewhat elusive in mainstream contexts. This is why Attainment Company's integration of the UDL framework with language-building objectives is so timely. Attainment has carefully revisited their curricula and have applied UDL guidelines with special emphasis on language acquisition to aid educators in supporting both student academic and language achievement. Directions and suggestions for connecting prior knowledge, scaffolding language supports, modeling, providing visual aids, and clear steps for embedding UDL have been enhanced in their new curricula. Even more exciting are the clear protocols for assisting educators in teaching and the plethora of resources provided for students. Through this curriculum series, Attainment ensures both equity and accessibility for diverse groups of students, including those with the most extensive support needs.

—Christopher J. Rivera, PhD  
Associate Professor, East Carolina University



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# COMPONENTS



## INSTRUCTOR'S GUIDE

The **Explore Geometry Instructor's Guide** contains *Getting Started*, *Topic*, and *Challenge* lessons to be used in conjunction with all 10 chapters of the Student Book. Using evidence-based practices, the Instructor's Guide provides effective teaching strategies for all students. It also features content-based activities that instructors can implement to increase student understanding and foster engagement, and tips for how to use the included protractor and coordinate plane.

## STUDENT BOOK

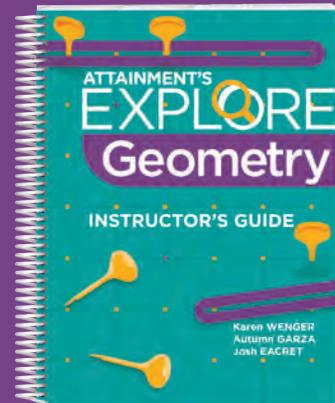
The **Explore Geometry Student Book** contains 10 chapters. Each chapter starts with a Big Picture statement, which gives an overview of the theme as it relates to an image from everyday life. Next, the four topics for the chapter are displayed with the topic illustration and the Remember! text. Each topic opens with an introductory statement from a professor, a topic-related illustration, and highlighted vocabulary that is explored within the text/dialog. After the instruction and example sections, there are two practice activities. These activities can be completed as a group or by each student individually. Following the four topics, a Chapter Challenge section introduces a final topic that builds upon the chapter skills, or introduces a related, advanced topic. Each chapter ends with a Chapter Check-up, consisting of a brief assessment from each of the chapter topics and challenge.

## DIGITAL LEVEL A

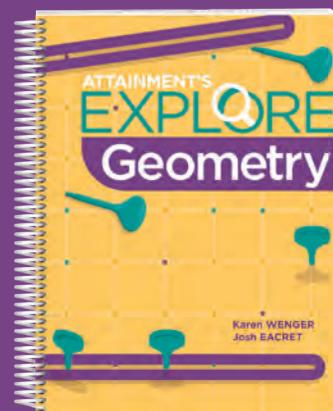
To differentiate lessons for varied levels of learners, we've created an Alternate level, **Level A**, for students who struggle to bridge concrete to abstract concepts. By lessening the complexity of the text and emphasizing the big ideas through illustrations, Level A provides a way for students with emerging skills to participate in upper-level geometry concepts. At the beginning of each lesson, teachers can choose between the printed or the alternate version (Level A) to adequately meet their students' needs. Students then show what they know by choosing the topic illustration that best depicts the lesson's big idea with each lesson closing. Additional tips for emerging learners are also provided in the *Foundational Concepts* section of the lessons. By providing differentiated levels, geometry is a subject area for all your students to explore!

## CONSUMABLE STUDENT WORKBOOKS

Two **Consumable Student Workbooks** correspond to the Student Book and reduce prep time. These workbooks offer students a workspace to write and record answers that can be kept and shared over time.



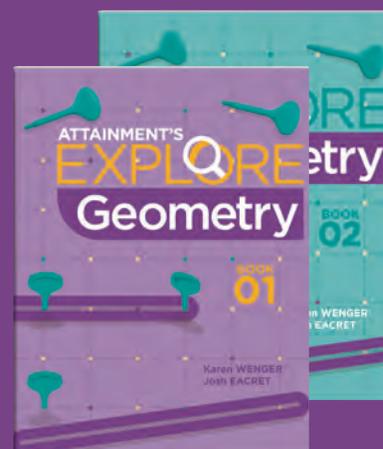
Instructor's Guide



Student Book



Level A



Student Workbooks 01 & 02

# COMPONENTS CONTINUED



## CARDS

**Vocabulary Cards** are provided for all vocabulary. These cards can be used with the Time-Delay strategy and Model-Lead-Test, as well as review activities. A **Time-Delay Procedure Card** featuring a templated script for vocabulary identification and definition comprehension is provided for you as a reference to use when teaching vocabulary terms.

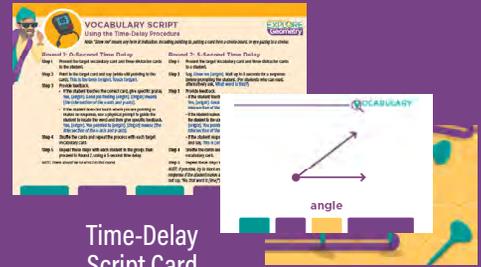
## MANIPULATIVES

An easy-to-read **Protractor** has been included to assist with finding angles. Most students will not be familiar with using a protractor, so practice will be essential.

A **Coordinate Plane** with coordinate pegs and a line indicator are also part of the Explore Geometry Curriculum. The coordinate plane is a handy graph with clearly defined x- and y-axes for quickly plotting points and lines that can be easily repositioned. Within the curriculum, the coordinate plane is useful for finding variables, plotting growth, tracking data, and measuring the slope of lines.

## DIGITAL FILES ON THE ATTAINMENT HUB

A variety of digital resources can be downloaded from the Attainment HUB for easy printing or projecting onto whiteboards. Extra activities, Chapter Check-Ups, and several blank templates for creating your own activities and tools are available. First, go to the Attainment HUB at <https://hub.attainmentcompany.com>. Follow the instructions provided on the inside front cover of the Instructor's Guide to redeem your HUB code and access all digital reproducible content and student reading materials.

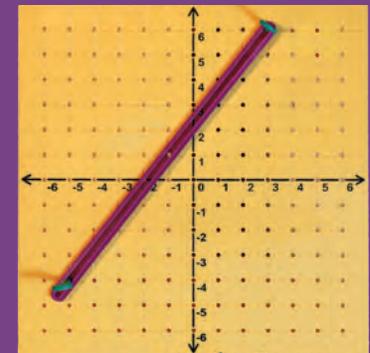


Time-Delay  
Script Card

Vocabulary Cards  
Samples



Protractor



Coordinate Plane



Digital Files are Available  
for Download via the HUB

# STARTING TO EXPLORE GEOMETRY



## LEVEL SELECTION

Before beginning Explore Geometry with students, assess their math skills to determine which level is most appropriate. The curriculum is designed to be used in conjunction with students at the standard and alternate level simultaneously as a means of differentiated instruction. Each lesson has suggestions for students with emerging skills within the *Foundational Concepts* area of the *Pre-Teach* section. The manipulatives can be used for both levels. You may find the manipulatives especially helpful with students in the Level A course of study for mastering early numeracy skills like rote counting, number identification, and basic math operations.

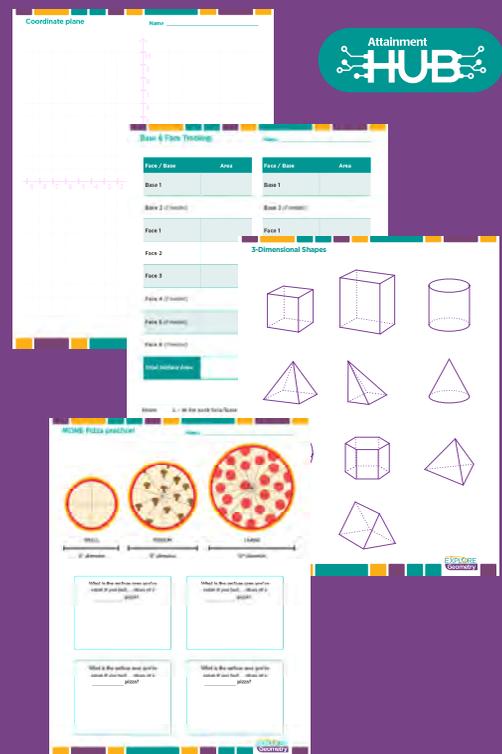
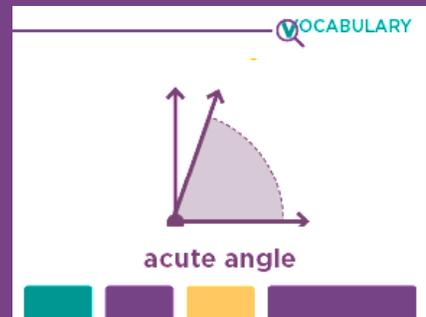
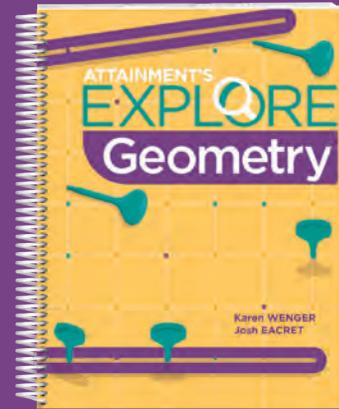
## VOCABULARY

You may find it necessary to substitute or add phrases that might clarify vocabulary words. For example, you may simplify the definition of *acute angle* to *the more closed angle*. As geometry terms are used multiple times throughout the curriculum, it is better that the student has a firm grasp on its meaning, rather than the printed definition.

*Pre-Teach.* Some terms may need to be explained before starting the actual lesson. Many of the topic introductions aim to make a connection between concrete and abstract concepts through real-life examples. By pre-teaching foundational math terms, students can focus on the lesson concepts.

## GRAPHIC ORGANIZERS

The use of graphic organizers have been shown to be an effective strategy that can be used across content areas for all students. There are many types of graphic organizers, but they all allow students to categorize information, making it less overwhelming and easier to manage. Throughout the Student Book, you will notice different ways of organizing information: number lines, coordinate grids, T-charts, Venn diagrams, etc. While it is suggested that the instructor presents the information within these graphic organizers, it is highly encouraged, when possible, that the students follow along and create their own graphic organizers (alone or with assistance), so that they have a better understanding of the building of the concepts. Instruction is short, but can be packed with steps and operations. It may be helpful to create a checklist for some more involved topics if you see that students are struggling to remember. Make it a group activity by having students identify each step!



# STARTING TO EXPLORE GEOMETRY CONTINUED

## MANIPULATIVES



Within general education, this course is typically taught by a credentialed math teacher. They often have a classroom with many different supports to assist in making abstract concepts more concrete. Many of the lessons are presented with these hands-on tools in mind. Specific examples for the set-up of angles and measurement are typically given within each lesson where it occurs. Similarly, a coordinate plane with easy-to-grasp pegs has been created and featured for many lessons on graphing. The larger lattice allows for easier counting of units for those with visual impairments. The sliding line piece makes it easy to identify other coordinates that are on the same linear path.

## CONCEPTS

The content of Explore Geometry assumes some background knowledge on the part of the students. Certain operations will be practiced, but it is necessary for students to understand basic math concepts in order to access later, more difficult content. Furthermore, geometry builds on itself. If a foundational concept is missed, or is not completely grasped, it's best to take extra class time to establish a solid understanding before moving into the lesson.

The content of Explore Geometry also assumes background knowledge on the part of the instructors. As most special educators do not have a math background, it will be very important to have a solid understanding of the content. Much of the "necessary foundational understanding" applies to instructors as much as students. You may want to review examples to ensure you can replicate them comfortably before the lesson. You may also choose to break certain "step-heavy" lessons into parts.

## UNIVERSAL DESIGN FOR LEARNING

Universal Design for Learning (UDL) is a framework to support and improve teaching and learning for all learners based on scientific findings (CAST; <https://www.cast.org/impact/universal-design-for-learning-udl>). This framework involves providing multiple means of *Representation*, *Expression*, and *Engagement*. Representation focuses on *how* information is presented. Expression focuses on varying ways that students can demonstrate understanding, and Engagement focuses on keeping students motivated and interested in learning. A general UDL table is provided to the right with some general suggestions for multiple means of Representation, Expression, and Engagement that are not content-specific and, therefore, applicable across lessons. *Additional, content-specific suggestions will be provided for all lessons in the curriculum.*

## ENGAGEMENT



- Assure students have background knowledge.
- Increase physical engagement by asking students to follow along as you read the text.
- Give students opportunities to participate in reading the text.
- Break up lessons into small increments.
- Differentiate the complexity of the text.
- Use the Student Book as is when appropriate.
- Lessen the complexity of words and phrases by substituting fewer complex words or adding explanations or details to more complex words or phrases.
- Communicate the lesson objectives in a meaningful manner.
- Offer explicit opportunities to generalize learning into new situations (*e.g., a real-world application*).
- Whenever possible, give students choices regarding what activity to complete and how the activity is completed.

## REPRESENTATION



- Project content to a Promethean board.
- Provide large photos representing concepts and vocabulary.
- Provide physical objects or special representations for concepts and vocabulary.
- Connect concepts and vocabulary to the learner's experiences and knowledge base.
- Add simulations, graphics, videos, and activities to concepts and vocabulary.
- Use strategically-placed symbol supports with concepts and vocabulary, for example.
- Provide options for organizing information.
- Repeat lessons, providing opportunities for review and practice.
- Enlarged text and/or images.
- Access to reference materials.

## EXPRESSION



- Provide assistive technology that is accessible for the student.
- Allow for the use of physical manipulatives and/or actions to demonstrate understanding.
- Provide preprogrammed AAC devices.
- Allow students to respond from an array of options.
- Add physical modifications to the Student Book to support students' ability to turn pages and locate chapters.
- Physical modes of responding may include pointing to, pulling off, or eye gazing to a selected choice.

UNIVERSAL DESIGN FOR LEARNING (UDL) FRAMEWORK

# EMBEDDING EVIDENCE-BASED TEACHING PROCEDURES

## MODEL-LEAD-TEST PROCEDURE (MLT)

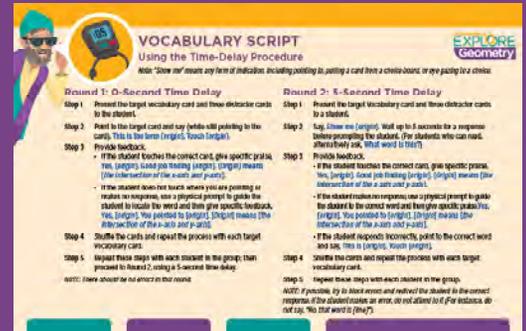
Evidence-based teaching procedures can effectively be utilized when completing the steps of each lesson. One procedure that can be embedded is the direct instruction method of MLT (Archer & Hughes, 2011; Bursuck & Damer, 2011). In MLT, you first model the target behavior (“Watch me”), then you lead the students to practice together with you in unison (“Do it with me”). After you have modeled the target behavior and led students through the process, the last step (test) requires the students to practice the target behavior independently (“Your turn”). The MLT is one procedure that is used for teaching vocabulary. While not scripted into the lessons, Model-Lead-Test is also appropriate for the first lesson in each chapter, which introduces the topics for the chapter. Keep in mind that MLT is a procedure that requires a verbal response from the student.

## TIME-DELAY PROCEDURE

Time-Delay is also a suggested evidence-based practice for teaching the geometry vocabulary in Explore Geometry. An example of the Time-Delay is as follows. The teacher lays out four vocabulary cards. She gives the instructions, “Point to acute angle,” and immediately points to the vocabulary card for “acute angle” so the student knows where to point. When the student consistently responds at zero-time delay (Round 1), the teacher gives the direction to the student but delays prompting for a specified number of seconds (e.g., 4–5 seconds) to provide the student the opportunity to respond independently (Round 2). Several questions should be considered for each student when using this procedure:

- What type of response does the student use: point to the answer (receptive)? Say the answer (expressive)? Pull the answer from a choice board? Eye gaze to an answer?
- Will the student respond receptively only or receptively and expressively (e.g., *point to the answer only; point to the answer and say it; use an AAC device to respond*)?
- How many warm-up trials will you give at 0-second time delay?
- How long will you wait before prompting in Round 2 (e.g., 5 seconds)?
- What kind of feedback will you provide? If a student struggles with Round 2, return to Round 1 for a few trials and then return to Round 2. For students who are consistently responding in Round 2, you can skip Round 1.

For your convenience, a Time-Delay Procedure card is provided. Vocabulary cards are provided with your kit for use during instruction.





# Chapter 2

## ANGLES AND TRIANGLES

Chapter 2 deals with geometry terminology, relationships between lines forming angles, and an introduction to types of triangles and their respective measures. It addresses the following standards:

**HSG.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

**HSG.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

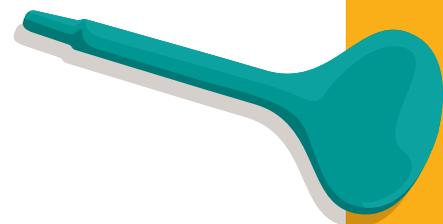
**HSG.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

**HSG.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**HSG.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**HSG.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

**HSG.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.



## PRE-TEACH



### I CAN STATEMENT(S)

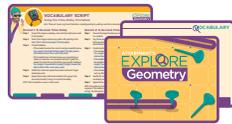
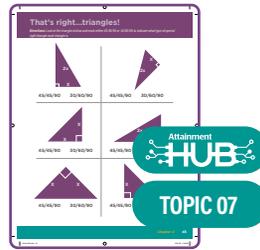
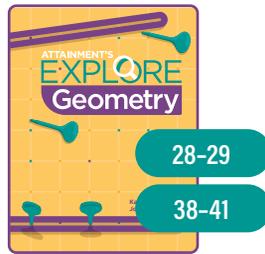
- 1 • I can use angles and sides to classify triangles.



I can select the topic illustration that depicts the big idea of the lesson.

### MATERIALS LIST

- Explore Geometry Student Book  
pgs. 28–29; 38–41
- Protractor
- Explore Geometry .pdf files  
from the HUB: Topic 07
- Lesson vocabulary cards:  
*congruent*
- Time-Delay Script Card
- Pieces of string at least 3 feet long



### ALIGNMENT

Prove geometric theorems.

**HSG.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Make geometric constructions.

**HSG.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

**HSG.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Prove theorems involving similarity.

**HSG.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

### FOUNDATIONAL CONCEPTS

Students should have an understanding of angle and angle classifications. The angle size is one way to group triangles. Additionally, the student should understand the term congruence for determining if angles or side lengths are the same. Markings used in geometry drawings for marking congruence should be reviewed or explained to students. Diagrams use a single hash line for indicating a side or angle are the same. If additional sides or angles are the same, a double hash line is used. To emphasize the same values for angles or sides, students could write the amounts on the diagrams.

For students with emerging skills, work on using the protractor to find prominent angles (e.g.,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ ), and basic math operations. You may also draw and label  $90^\circ$  and  $45^\circ$  angles so students become accustomed to approximation of angles.

## ENGAGEMENT



- Enter *Classifying Triangles Song* into your preferred Internet search engine and view the results. Get students excited about the triangle types!
- As a group, create a Frayer model graphic organizer to develop student understanding of the vocabulary term *congruence* or *congruent*. Discuss the definitions, characteristics, and examples/non-examples of the word.
- Enter *free printable triangle types* into your preferred Internet search engine. Cut out the different triangle types. Have students select a triangle, and then identify it by name.

## REPRESENTATION



- Create a graphic organizer in which you can sort the different types of triangles (e.g., *scalene*, *isosceles*, *equilateral*, *obtuse*, *right*, *acute*, or *equiangular*) by sides or by angles.
- Show real-world examples of the different triangle types and project them onto a whiteboard. The Bermuda Triangle, pyramids, traffic signs, and roofs are just a few examples.
- Review Topic 5 on *Angles* to ensure students understand the different angle types. This will help them as they work on identifying the triangle types.

## EXPRESSION



- Have students declare one fact about a triangle type. Preprogram the statements for students using AAC devices. For example, *a right triangle has one right angle*.
- Ask students to choose their favorite real-world example of a triangle. Have them share their selection with the group and explain how its features tell us the triangle type.
- Allow students to use a YES/NO response with an AAC device or YES/NO response cards as an alternative to using an array of response options (e.g., *Is this an isosceles triangle?*).

UNIVERSAL DESIGN FOR LEARNING (UDL) FRAMEWORK

# TEACH



## ATTENTION GETTER / INTRODUCE THE LESSON



If you remember, our last few topics have been about the different ways we can identify angles. Who can remember some angle types? Accept answers. **Very good!** List some of the angle types. **Those are types of angles! We're going to keep using some of those terms and also learn some new terms!**

Have the students turn to pg. 38 in the Student Book. **Today, we will begin *Triangle types*.**

**Professor Maryam says, "After looking at all of these angles, I think we're ready to try triangle angles!" Who can tell me about triangles?** Have students make suggestions. **That's right, triangles have three sides and 3 angles.**

## TOPIC VOCABULARY



Choose the best strategy for your students:



### Model-Lead-Test

**Model:** The term [*congruent*] means [*when an angle, side, or shape is the same shape, size, or measurement; equal*].

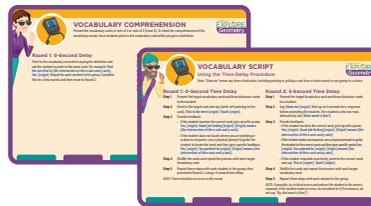
**Lead:** Say it with me. [*Congruent*] means [*when an angle, side, or shape is the same shape, size, or measurement; equal*].

**Test:** What term means [*when an angle, side, or shape is the same shape, size, or measurement; equal*]?

Student indicates correct vocabulary word. Continue with any additional vocabulary words.

### Time-Delay

Write out the vocabulary so that it can be easily displayed. Follow the Time-Delay procedure. Repeat the procedure for all vocabulary words in the lesson.



## LESSON WALK THROUGH



Choose between the printed or the alternate version of the lesson. **Let's start reading about today's topic.**



**A triangle is any three, nonlinear points on a coordinate plane that are joined. *Nonlinear* means *not in a line*. Let's see what that means.** Draw three points on the whiteboard, then connect the points to form a triangle. **Every triangle has two key features: It has three sides.** Count the sides of the drawn triangle aloud. **Check! And, it has three vertices.** Count each vertex of the triangle aloud. **Check! Looks like we have a triangle here. Lastly, the sum of the three, inside angles of any triangle, point to each of the three interior angles of the triangle, is equal to 180°! Isn't that cool?! Let's take a look at some different triangles and see how they compare.** Look at each triangle on page 38 as a group. Write the measurements stacked, so you can add them, on the whiteboard. Show that the three measurements add up to 180.



## Tips for Teachers

Display pictures of a slice of pizza, a piece of pie, an apple wedge, and an orange slice (try and show the top-down view of the orange and apple so students can see the triangular shape).

## VOCABULARY

### Congruent

When an angle, side, or shape is the same shape, size, or measurement; equal.

### Terms to Know

#### Scalene triangle

A triangle where no sides are equal; all sides are different.

#### Isosceles triangle

A triangle where at least two sides are equal.

#### Equilateral triangle

A triangle where all sides are equal.

**TOPIC 7 Triangle types**

After looking at all of these angles, I think we're ready to try triangle angles!

A triangle is any three, nonlinear points on a coordinate plane that are joined.

Every triangle has two key features:

- It has three sides.
- It has three vertices.

THREE SIDES, THREE VERICES, AND THE ANGLES ADD UP TO 180° ALL THE TIMES!! Isn't that cool?!

**REMEMBER!** Triangles are formed from the joining of three, nonlinear points.

Let's take a look at some different triangles and see how they compare.

180°

180°

180°

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## LESSON WALK THROUGH CONTINUED



Present the Fun Fact. Encourage anticipatory discussion based on the Fun Fact. **Here's a Fun Fact: The Triangle Sum Theorem says that the sum of the angles of any triangle always equals 180°.**

**Triangles, like angles, can also be identified by different angle measurements. However, UNLIKE angles, triangles can also be identified by the congruence of their sides.**

**One way to classify triangles is by sides. We look at the sides and see if the length is equal to another side. Scalene triangle: no sides congruent. Isosceles triangle: at least two sides congruent. Equilateral triangle: all three sides congruent.** If possible, use a yardstick to measure out/draw triangles with zero, two, and all three sides congruent.

**Another way to classify triangles is by angles: Obtuse triangle: one obtuse angle. Right triangle: one right angle. Acute triangle: three acute angles. Equiangular triangle: all three angles congruent.** For each triangle type, draw the corresponding angle.

## REINFORCEMENT • REMEMBER!

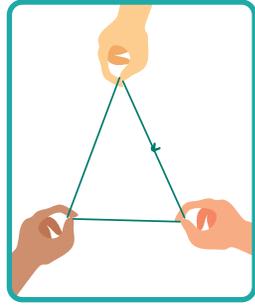


Read the Remember! text aloud and have students read along with you. **Triangles are formed from the joining of three, nonlinear points.** Repeat and leave blanks for students to fill in the missing words aloud. Reinforce and confirm correct responses. If needed, re-read portions of the text to emphasize the correct answer.

## DISCUSSION



Start a discussion by saying, **When we say that a triangle is formed from three, nonlinear points, we don't really mean ANY three points, do we? Is it really possible to make a triangle with any three points?** Accept answers. **Let's find out!** Divide students into groups of at least 3. Have some lengths of string ready; tie the ends of each piece of string together so that you have loops. Have 3 students in each group hook a finger around the string and spread out so that the string is taut. No matter where they move, the three students will form a triangle with the string. At regular intervals, have the students FREEZE! and students not holding the string can draw the triangle that has been created.



As a language-building activity, students can create a poster of their various triangles and then label them by the types of triangles created. Have students create a graph of the frequency of triangle types and present it to the class using pertinent lesson vocabulary.

## APPLY



### MORE TRIANGLE TYPES



**On the next page, we're going to practice identifying triangles. Let's read the directions and complete our first activity.**

Once the page has been completed, have students present their answers to each other or to you in front of their peers. Reinforce and confirm correct answers. For any correction, discuss the answer aloud as a group. If students are struggling, provide more problems on the whiteboard and solve them as a group.

**Fun Fact:** The Triangle Sum Theorem says that the sum of the angles of any triangle always equals 180°.

Triangles, like angles, can also be identified by different angle measurements. However, UNLIKE angles, triangles can also be identified by the congruence of their sides.

**Triangle Classification by Sides**

- Scalene triangle: no sides congruent
- Isosceles triangle: at least two sides congruent
- Equilateral triangle: all three sides congruent

**Triangle Classification by Angles**

- Obtuse triangle: one obtuse angle
- Right triangle: one right angle
- Acute triangle: three acute angles
- Equiangular triangle: all three angles congruent

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## REMEMBER!

Triangles are formed from the joining of three, nonlinear points.



## Tips for Teachers

For a fun, out-of-classroom activity, go to the gymnasium so students can be up higher. Using a very long, tied rope or cord, have students move around and form gym-sized triangles while the viewing students draw each triangle formed.

Students can also instruct the "vertex" students to move to different locations to create different triangle types!

**More triangle types**

Directions: Look at the figures and their measurements below. Then, use the word bank to write what type of triangle each shape might be. There may be more than one match for each triangle.

Scalene	Obtuse	Acute
Isosceles	Right	Equilateral

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## TRY-ANGLES

2

On the last page of this topic, we're going to keep identifying triangles! Let's read the directions and complete our second activity.

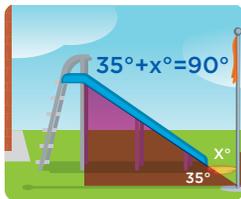
Once the page has been completed either as a group or individually, have students present their answers to each other or to you in front of the group as another language-building exercise.

## CLOSING

Review the topic by doing one or more of the following:



- Read the topic from the first page;
- Read, or read along with, the Remember! text;
- Select the current topic from a group of topics;
- Select the topic illustration from a group of illustrations.



## EXTENDED PRACTICE



For extended practice in creating and identifying triangles, use dice\*\* or a spinner with numbers and have students use random numbers for lengths of sides. With a ruler, students should draw two sides of a triangle (using the random measurements as lengths.); then, connect the two segments to complete the triangle. Students can then write the triangle type(s) next to the drawing.



As already noted, some triangles fit into multiple types of classifications/types. For example, *an isosceles triangle is also an acute triangle*. As a language-building activity, have students research other types of classifications and things that can fit into multiple categories. Students can then present their findings to the group.

## EXTEND



### CHALLENGE EXTENSIONS



As a challenge, use the coordinate plane to graph lines of given lengths, trying to form the lines into a triangle:

- |               |                            |                             |
|---------------|----------------------------|-----------------------------|
| 4, 5, 7 units | 3, 5, 10 units             | 76°, 34°, length of 9 units |
| 3, 4, 5 units | 6 & 10 units, what is the  | between them                |
| 3, 3, 3 units | shortest the third can be? | 30°, 8, 4 units             |

Additional practice sheets may be accessed and printed from the HUB.



**Try-angles!**  
Directions: Match the triangle type in the left column to the triangle in the right column. There may be more than one match for each triangle type or triangle.

Equilateral	
Scalene	
Isosceles	
Obtuse	
Acute	
Right	

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\*\*Digital dice or spinners can be found on online for free by searching "virtual dice/spinner" in your preferred Internet search engine.

An equilateral/equiangular triangle will fit inside a circle and touch at 3, evenly spaced points. This is an easy way for students to identify those types of triangles.

Let's Practice! TOPIC 07

Directions: Match the triangle type in the left column to the triangle in the right column. There may be more than one match for each triangle type. Draw a line to connect the triangle to its type.

Acute	
Obtuse	
Triangle	
Equilateral	

EXPLORE CONNECTIONS